Thursday, June 8, 2000 11:10 AM - 12:40 PM

i.) This test is closed book.
ii.) There are 7 questions. Answer all questions.
iii.) Note problems are NOT of equal value.

1.) (10 pts.) A passive scalar of concentration c is mixed into a turbulent flow well-described by K41. Assuming the passive scalar has a decay rate $\alpha = d\left(c^2\right)/dt$, and that the flow has dissipation rate ε, calculate:

a.) the spectrum of concentration fluctuations,
b.) the dissipation scale for concentration fluctuations, assuming c has diffusivity D.

2.) (10 pts.) State Kolmogorov's 4/5 Law and discuss it in a short essay. Your discussion should address the following points:

a.) why 4/5 Law is important,
b.) why $S_3(\ell) \neq 0$, and what it implies about RPA-based closure theories,
c.) significance of 4/5 Law for intermittency models,
d.) a brief sketch of how you would set-up proof of 4/5 Law.

3.) (10 pts.) Consider a turbulent pipe flow. Derive the Prandtl 'Law of the Wall', using mixing length theory. Carefully state all assumptions. Identify all sublayers and sublayer widths. Assume the flow exerts wall stress $\rho_0 \sigma$.

4.) (10 pts.) Derive the renormalized propagator for 1D Burgers turbulence, governed by:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} - \nu\frac{\partial^2 v}{\partial x^2} = f$$

where f is a random force. Determine the eddy viscosity and how it scales with fluctuation intensity. Explain the role of the eddy viscosity in a renormalized spectral equation.

5.) (10 pts.) Present an argument as to why 2D hydrodynamic turbulence exhibits a dual cascade. Use any and all theoretical arguments at your disposal.

6.) (20 pts.) a.) Derive the Hasegawa-Mima equation for potential fluctuations in drift-wave turbulence, starting from the cold ion fluid equations and the assumption that $\tilde{n}/n = |e|\tilde{\phi}/T$.

b.) Use closure theory to carefully discuss the distinction between 'weak' and 'strong' turbulence regimes in drift wave turbulence.

7.) (30 pts.) a.) Briefly explain the problem of intermittency as it is manifested in the behavior of higher order moments $\left(S_p(\ell),\ \text{for } p > 3\right)$ of the velocity increment $\delta v(\ell)$.

b.) Use the beta-model to estimate $S_p(\ell_d)$ for $p = 4$. Make sure your result is consistent with the 4/5 Law.

c.) Briefly explain the significance of a bi-fractal or multi-fractal model of intermittency in turbulence. What aspect of the data makes such models interesting?